Polar Graphing Guide

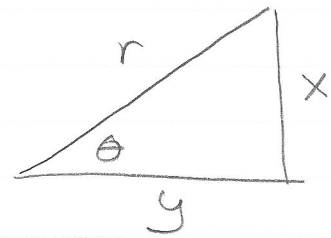
to help with 10.3

Some hints and observations

basic polar coordinate definitions

$x = r\cos\theta$

$y = r\sin\theta$



so $\boxed{r\cos\theta = a} \Rightarrow \boxed{x = a} \Rightarrow$ vertical line

$\boxed{r\sin\theta = b} \Rightarrow \boxed{y = b} \Rightarrow$ horizontal line

$\boxed{r = \pm 2a\cos\theta}$

circle of radius "a" to the right or left of the origin

+R
−L

$\boxed{r = \pm 2a\sin\theta}$

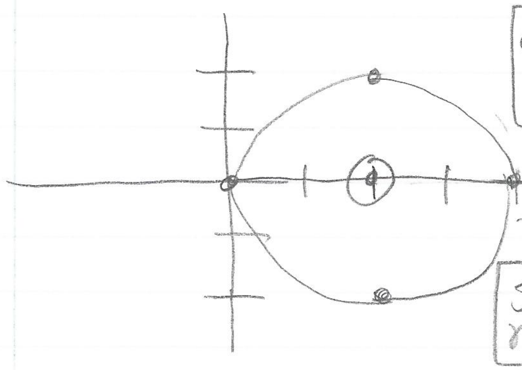
circle of radius "a" above or below the origin

+ up
− down

$\boxed{r = 4\cos\theta}$

$2a = 4$

$a = 2$ (the radius)



center (2,0)

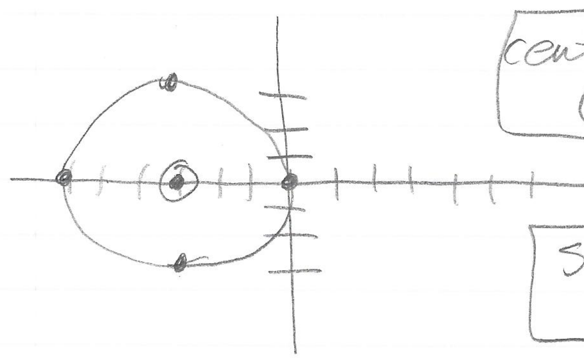
shifts right

from 0 to 2π

$\boxed{r = -6\cos\theta}$

$2a = 6$

$a = 3$ (the radius)



center (−3,0)

shifts left

from 0 to 2π

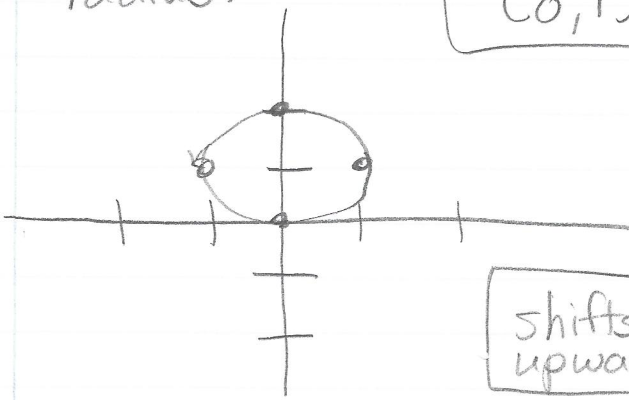
$+ \rightarrow$ up

$\boxed{r = 2\sin\theta}$

$2a = 2$

$\boxed{a = 1}$ radius.

center $(0,1)$



shifts upward

from 0 to 2π

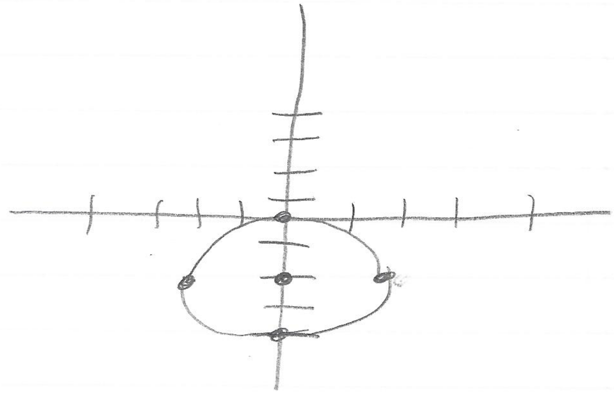
down

$\boxed{r = -4\sin\theta}$

$2a = 4$

$\boxed{a = 2}$ radius

center $(0,-2)$



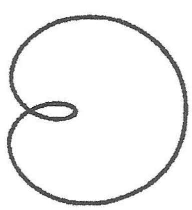
from 0 to 2π

Limacons $r = a \pm b\sin\theta$ / $r = a \pm b\cos\theta$

Limacons

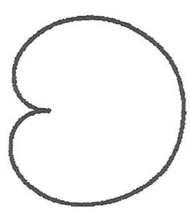
$r = a \pm b\sin\theta$ or $r = a \pm b\cos\theta$

$a/b < 1$



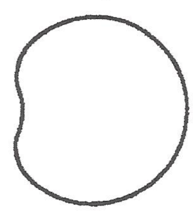
Limacon with Inner Loop

$a/b = 1$



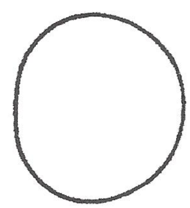
Cardioid

$1 < a/b < 2$



Dimpled Limacon

$a/b \geq 2$

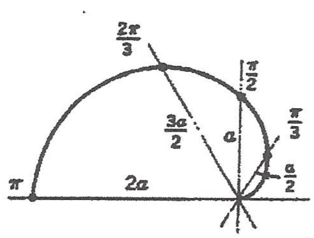


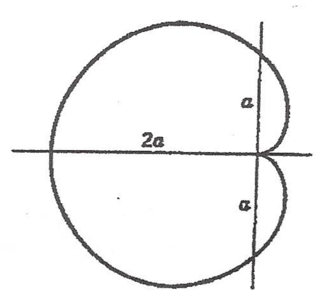
Convex Limacon

Consider:

$a = b \Rightarrow$ Cardioid

$r = a(1 - \cos\theta)$



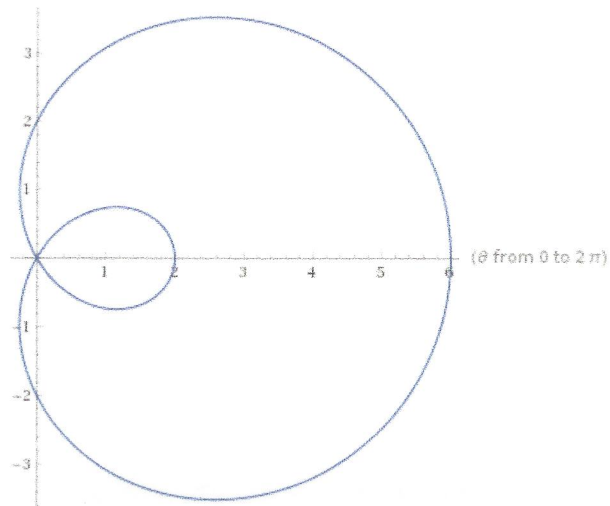


First Let's explore what happens when the signs change and the difference between sin and cos graphs ③

$$\boxed{r = 2 + 4\cos\theta}$$

plot $r = 2 + 4\cos(\theta)$

Polar plot:



Recall

$$\boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}}$$

+ ×

$$\boxed{r = 2 + 4\cos\theta}$$

+ x-axis

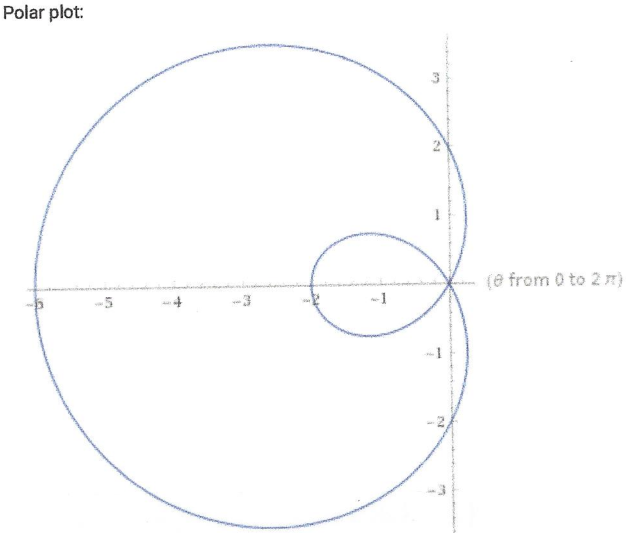
$\frac{2}{4} = \frac{1}{2} < 1$

$\Rightarrow$ loop

$$\boxed{r = 2 - 4\cos\theta}$$

plot $r = 2 - 4\cos(\theta)$

Polar plot:



− ×

$$\boxed{r = 2 - 4\cos\theta}$$

− x-axis

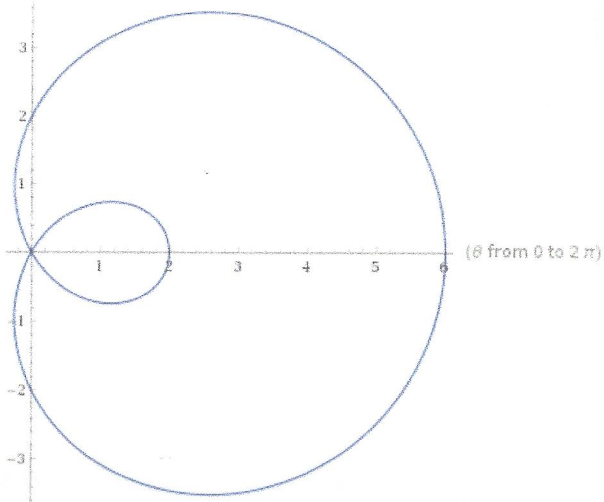
Recall

$x = r\cos\theta$
$y = r\sin\theta$

$r = -2 + 4\cos\theta$

plot $r = -2 + 4\cos(\theta)$

Polar plot:



+ x

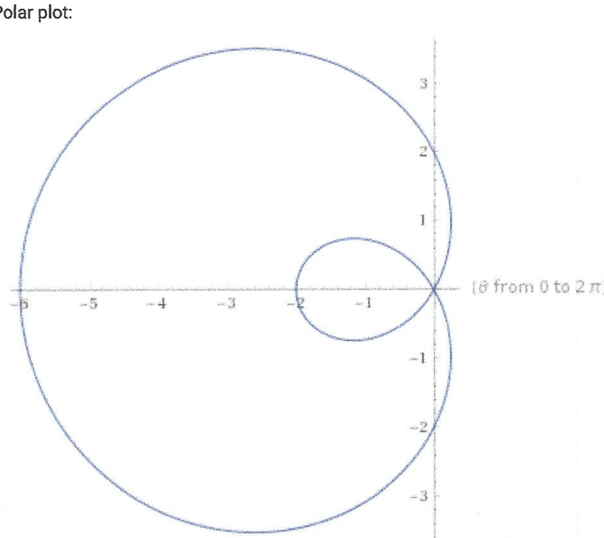
$r = -2 + 4\cos\theta$

+ x-axis

$r = -2 - 4\cos\theta$

plot $r = -2 - 4\cos(\theta)$

Polar plot:



− x

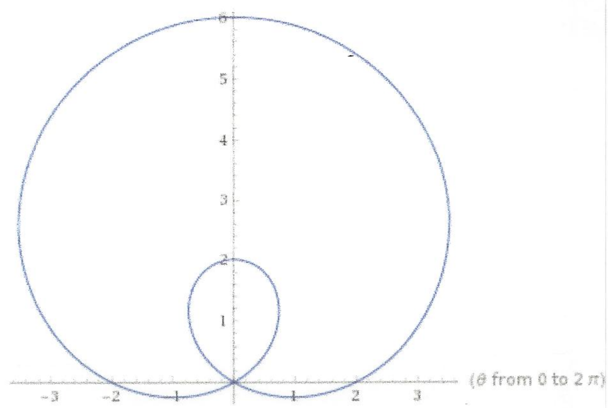
$r = -2 - 4\cos\theta$

− x-axis

$\boxed{r = 2 + 4\sin\theta}$

plot $r = 2 + 4\sin(\theta)$

Polar plot:



Recall

$$\boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}}$$

+ y

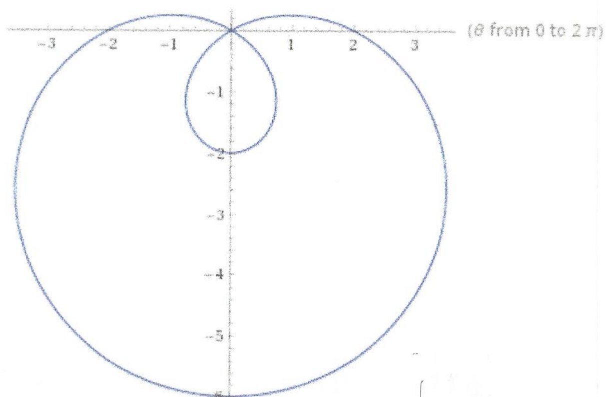
$\boxed{r = 2 + 4\sin\theta}$

+y axis

$\boxed{r = 2 - 4\sin\theta}$

plot $r = 2 - 4\sin(\theta)$

Polar plot:



− y

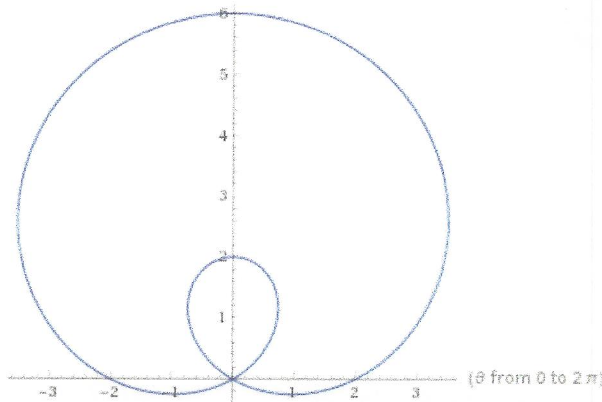
$\boxed{r = 2 - 4\sin\theta}$

− y-axis

$r = -2 + 4\sin\theta$

plot $r = -2 + 4\sin(\theta)$

Polar plot:



$r = \underset{+}{-2} + \underset{+\,y\text{-axis}}{4\sin\theta}$

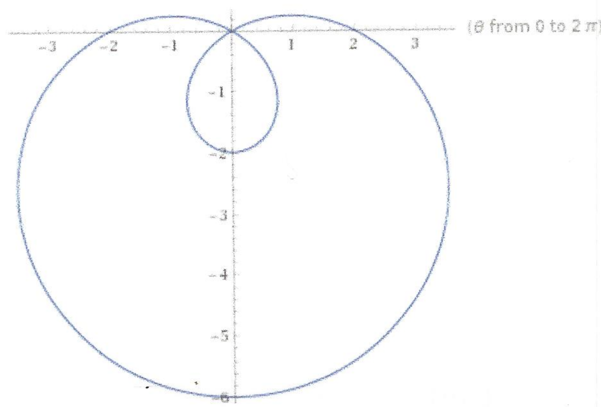
+ y

+y-axis

$r = -2 - 4\sin\theta$

plot $r = -2 - 4\sin(\theta)$

Polar plot:



$r = -2 - 4\sin\theta$

− y

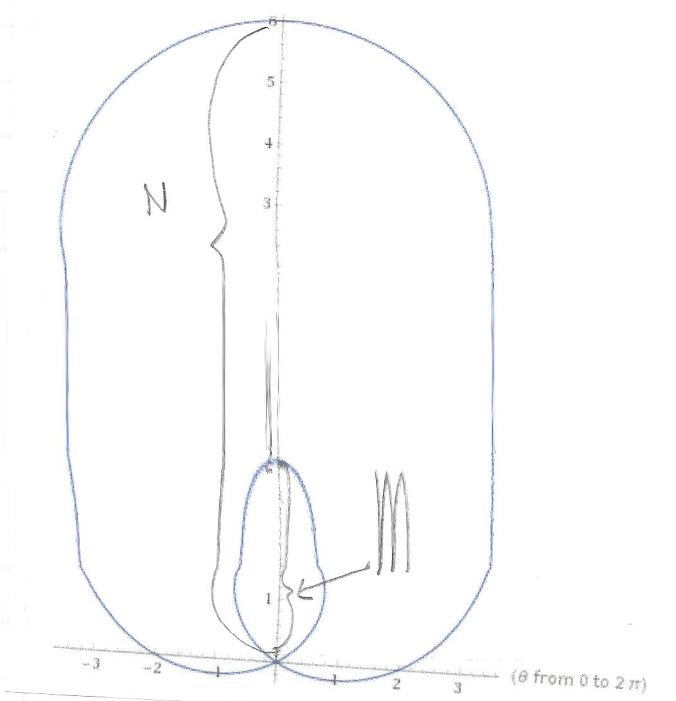
−y-axis

note that these graphs are the same

$r = 2 + 4\sin\theta$

plot $r = 2 + 4\sin(\theta)$

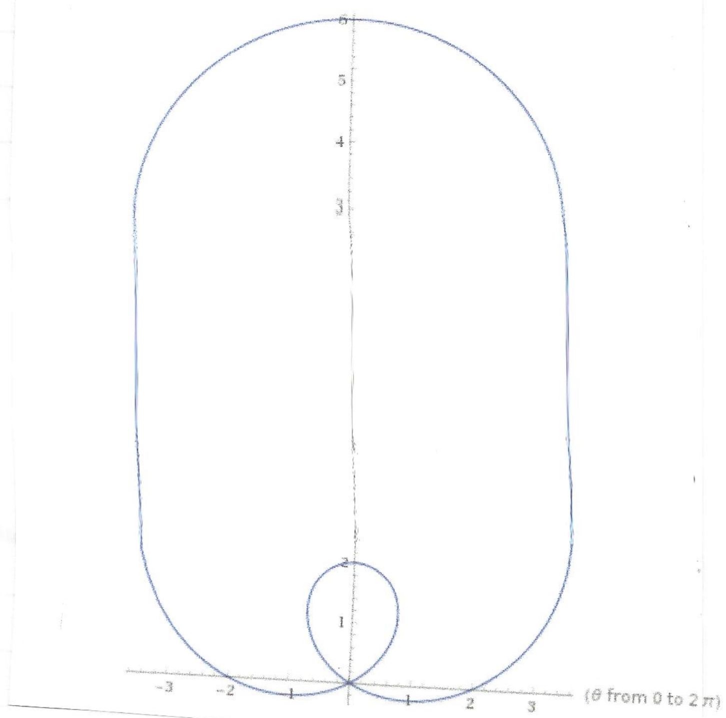
Polar plot:



$r = -2 + 4\sin\theta$

plot $r = -2 + 4\sin(\theta)$

Polar plot:



$\boxed{\begin{array}{c} a = 2 \\ b = 4 \end{array}}$

so $r = 2 + 4\sin\theta$

$\boxed{\begin{array}{c} a = -2 \\ b = 4 \end{array}}$

$r = -2 + 4\sin\theta$

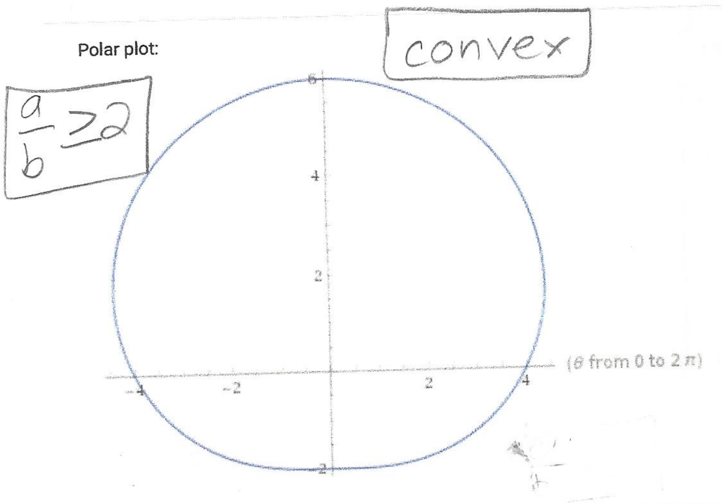
so the sign of a <u>does not</u> affect the axis location

the sign of b <u>does</u> affect the axis location (direction)

more about Limacons

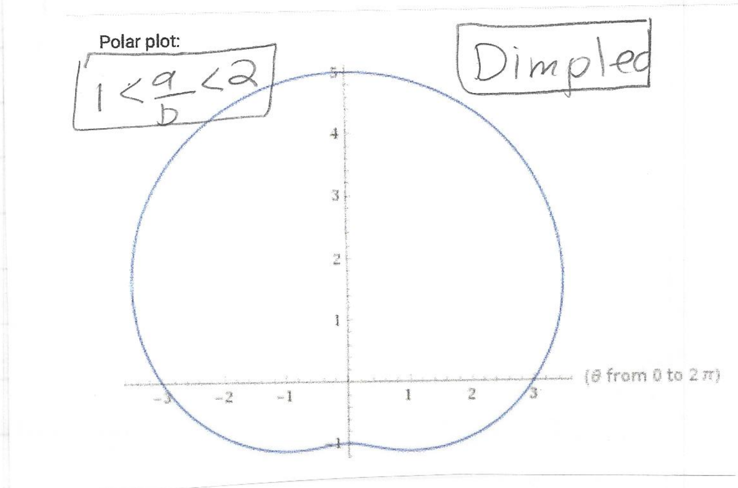
Now let's explore what happens when we adjust a & b.

plot $r = 4 + 2\sin(\theta)$



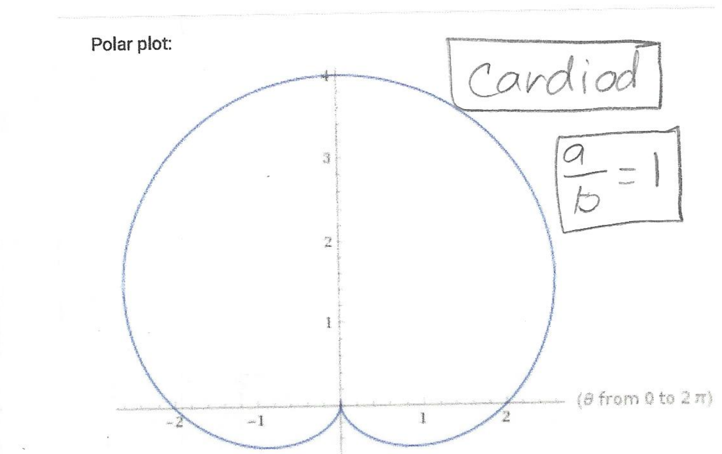
$\boxed{\frac{a}{b}} = \frac{4}{2} = \boxed{2} \geq 2$

plot $r = 3 + 2\sin(\theta)$



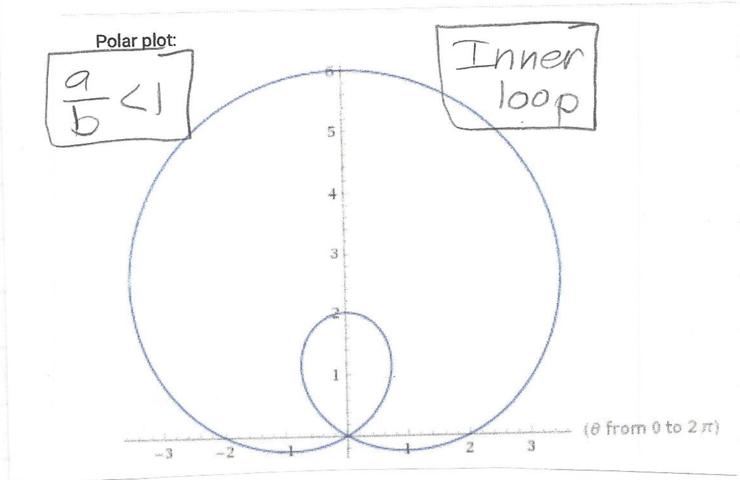
$\boxed{\frac{a}{b}} = \frac{3}{2} = \boxed{1.5}$

plot $r = 2 + 2\sin(\theta)$



$\boxed{\frac{a}{b}} = \frac{2}{2} = \boxed{1}$

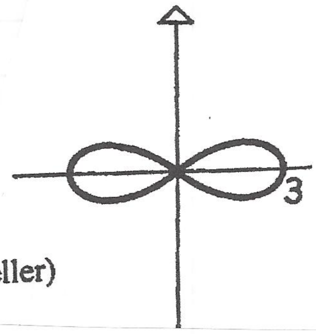
plot $r = 2 + 4\sin(\theta)$



$\boxed{\frac{a}{b}} = \frac{2}{4} = \boxed{\frac{1}{2}}$

Lemniscates (propellers)

$$r^2 = a^2 \cos 2\theta \quad \text{or} \quad r^2 = -a^2 \cos 2\theta$$
$$r^2 = a^2 \sin 2\theta \quad \text{or} \quad r^2 = -a^2 \sin 2\theta$$



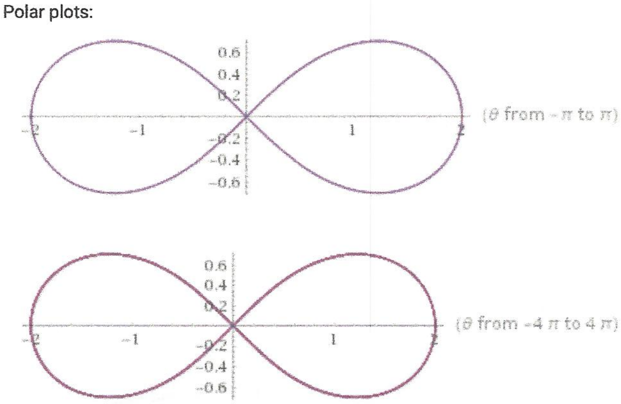
Lemniscate (propeller)

Due the r^2, we have to adjust our logic.

$$\boxed{r^2 = 4\cos 2\theta}$$

plot $r^2 = 4\cos(2\theta)$

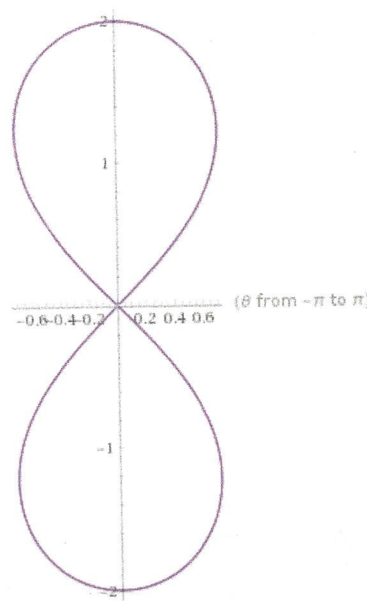
Polar plots:



$$\boxed{r^2 = -4\cos 2\theta}$$

plot $r^2 = -4\cos(2\theta)$

Polar plots:



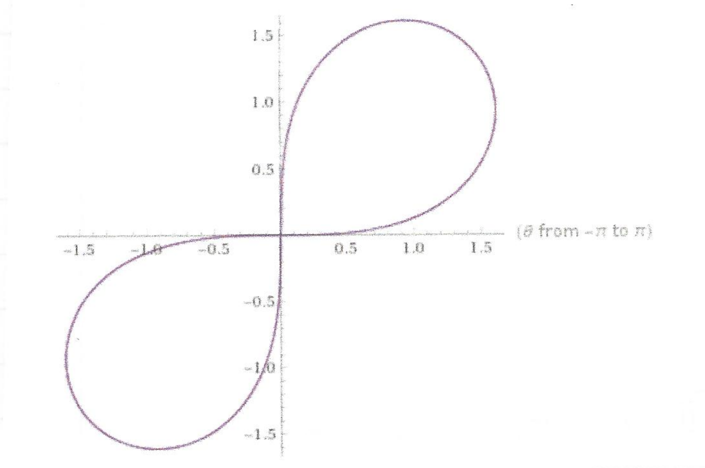
$\boxed{+\cos}$ → end of propellors on x-axis

$-\cos$ | end of propellors on y-axis

$\boxed{r^2 = 4\sin 2\theta}$ $\boxed{r^2 = -4\sin 2\theta}$

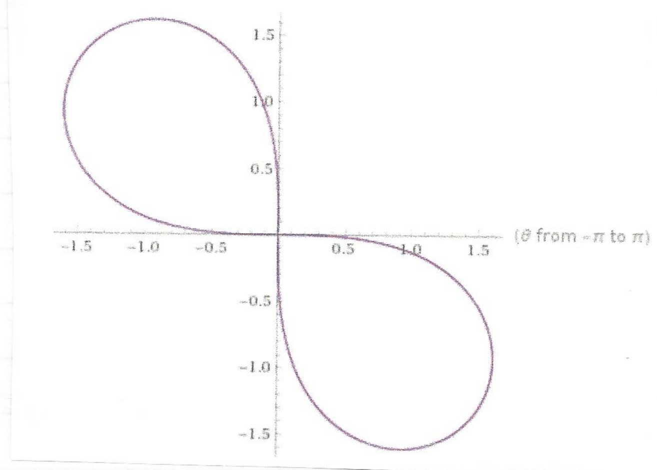
plot $r^2 = 4\sin(2\theta)$

Polar plots:



plot $r^2 = -4\sin(2\theta)$

Polar plots:



$\boxed{+\sin 2\theta}$

end of propellers on line $y = x$

$\boxed{-\sin 2\theta}$

$\boxed{\text{end of propellers on line } y = -x}$

$\sin\theta = 1$

$\Rightarrow \theta = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}, \ldots$

$2\theta = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}, \ldots$

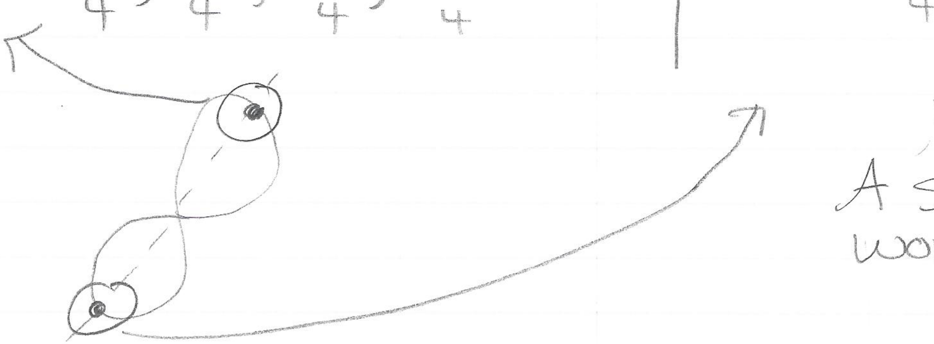
$\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}, \ldots$

$\sin\theta = -1$

$\Rightarrow \theta = \frac{3\pi}{2}, \frac{7\pi}{2}, \ldots$

$2\theta = \frac{3\pi}{2}, \frac{7\pi}{2}, \ldots$

$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}, \ldots$



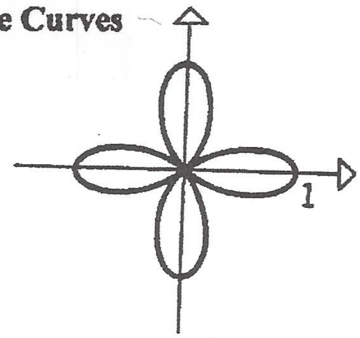
A similar process works for $\cos(2\theta)$

Rose Curves

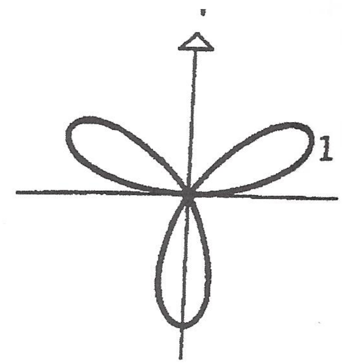
Rose Curves

$r = a\sin(n\theta)$ or $r = a\cos(n\theta)$

If n is odd, then the curve has "n" equally spaced petals.
If n is even, then the curve has "2n" equally spaced petals.



Four-petal rose



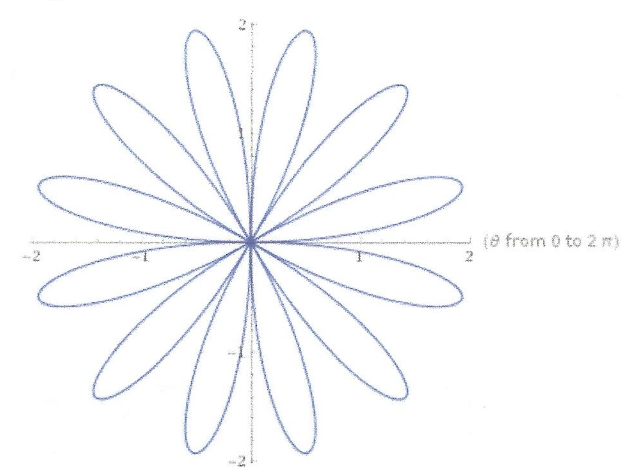
Three-petal rose

sin/even

$r = 2\sin 6\theta$

plot $r = 2\sin(6\theta)$

Polar plot:

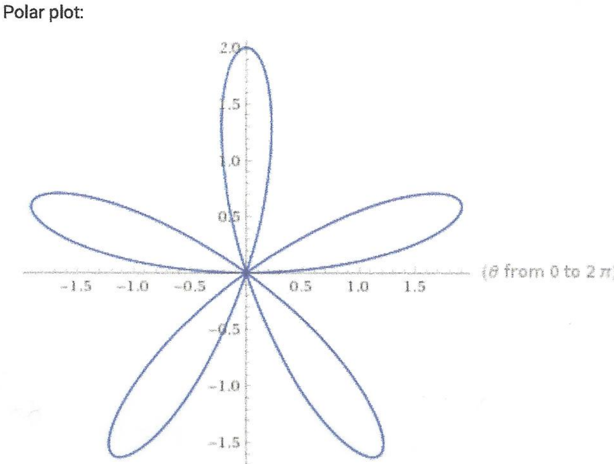


sin/odd

$r = 2\sin 5\theta$

plot $r = 2\sin(5\theta)$

Polar plot:



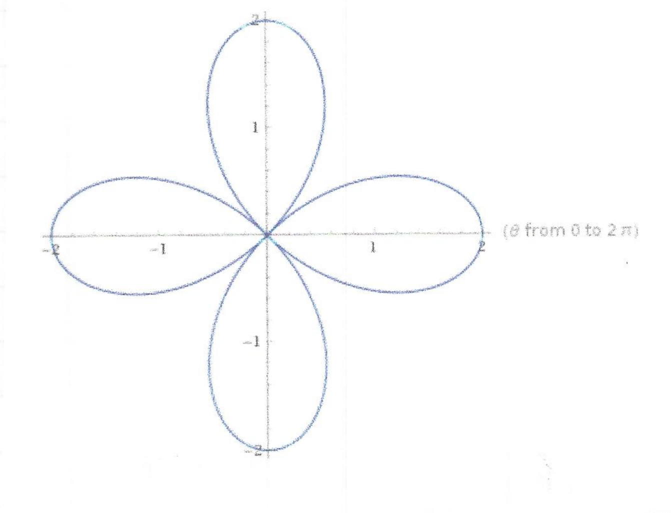
n=6 (even) 2n loops 12

n=5 (odd) n loops 5

cos / even	cos / odd
$r = 2\cos(2\theta)$	$r = 2\cos(5\theta)$

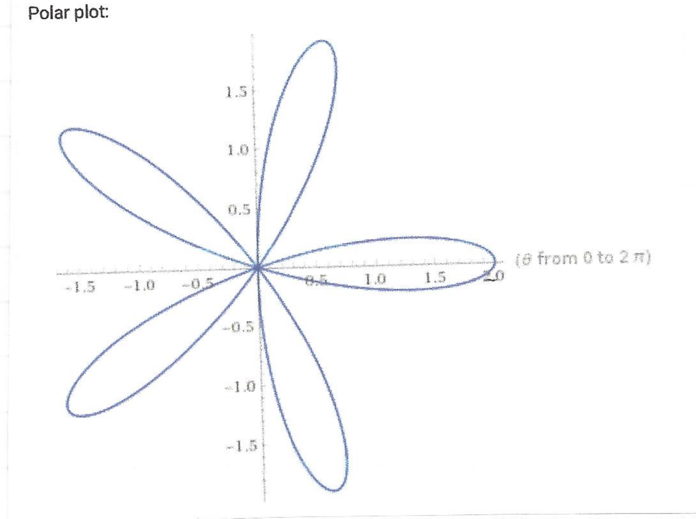
plot $r = 2\cos(2\theta)$

Polar plot:



plot $r = 2\cos(5\theta)$

Polar plot:



$\boxed{n=2}$ (even) — $2n$ loops: 4

$\boxed{n=5}$ (odd) — n loops: 5

The tips of the petals will be in different locations depending on whether the graph is a sin or a cos graph.

The example on the next page explores (2θ)

The general approach is:

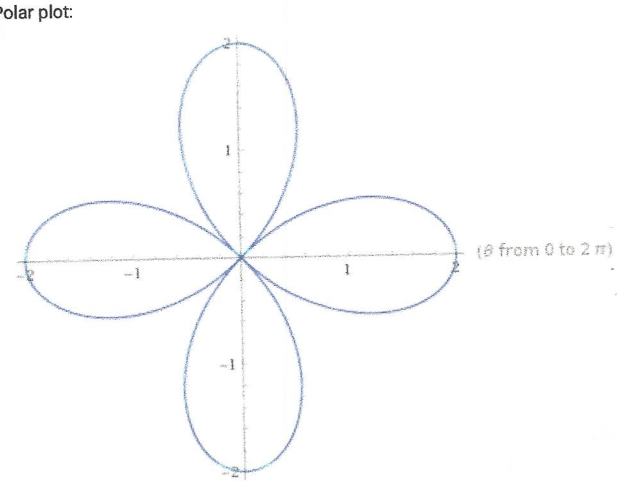
$$n\theta = A, B, C, \ldots$$
$$\theta = \frac{A}{n}, \frac{B}{n}, \frac{C}{n}, \ldots$$

How do sine & cosine graphs differ for Roses?

$\boxed{r = 2\cos(2\theta)}$

plot $r = 2\cos(2\theta)$

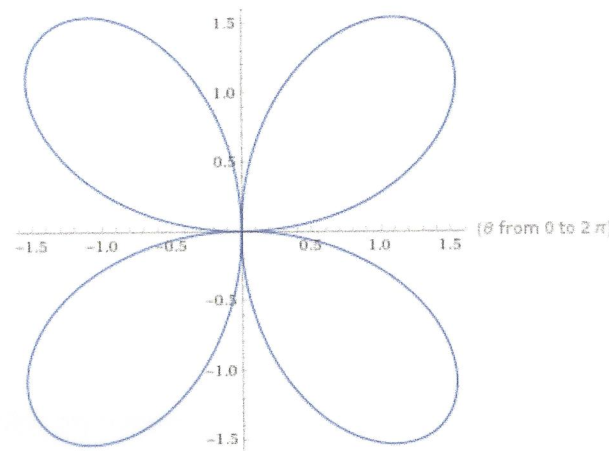
Polar plot:



$\boxed{r = 2\sin(2\theta)}$

plot $r = 2\sin(2\theta)$

Polar plot:



for cos

$\boxed{\cos\theta = 1}$

max occurs

$\theta = 0, 2\pi, 4\pi, \ldots$

min occurs

$\theta = \pi, 3\pi, 5\pi, \ldots$

$\boxed{\cos\theta = -1}$

zeros

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

for sin

$\boxed{\sin\theta = 1}$

max occurs

$\theta = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$

min occurs

$\theta = \frac{3\pi}{2}, \frac{7\pi}{2}, \ldots$

$\boxed{\sin\theta = -1}$

zeros

$\theta = 0, \pi, 2\pi, \ldots$

We use this information to help us when we are graphing.

Graphing Polar Curves

$r \cos \theta = a \rightarrow x = a \rightarrow$ vertical line $r \sin \theta = b \rightarrow y = b \rightarrow$ horizontal line

$r = a$, is a circle centered at the origin. $\theta = a$, is an angled line.

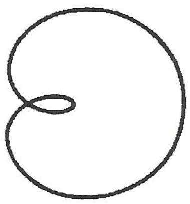
$r = \pm\, 2a \cos \theta$, is a circle of radius "a" right or left of the origin.

$r = \pm\, 2a \sin \theta$, is a circle of radius "a" above or below the origin.

Limacons

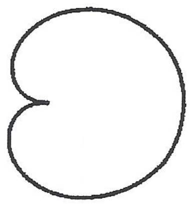
$r = a \pm b \sin \theta$ or $r = a \pm b \cos \theta$

$a/b < 1$



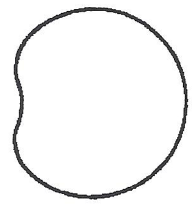
Limacon
with Inner Loop

$a/b = 1$



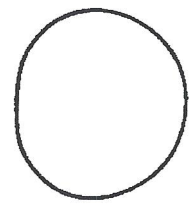
Cardioid

$1 < a/b < 2$



Dimpled
Limacon

$a/b \geq 2$

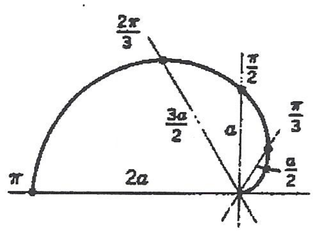


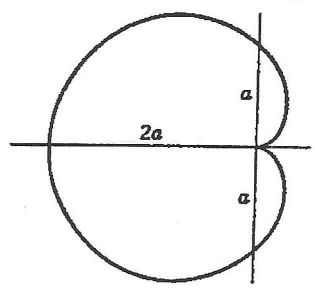
Convex
Limacon

Consider:

$a = b \rightarrow$ Cardioid

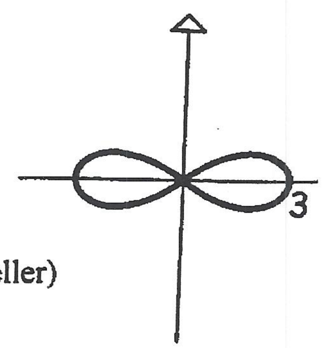
$r = a\,(1 - \cos \theta)$





$r^2 = a^2 \cos 2\theta$ or $r^2 = -a^2 \cos 2\theta$

$r^2 = a^2 \sin 2\theta$ or $r^2 = -a^2 \sin 2\theta$

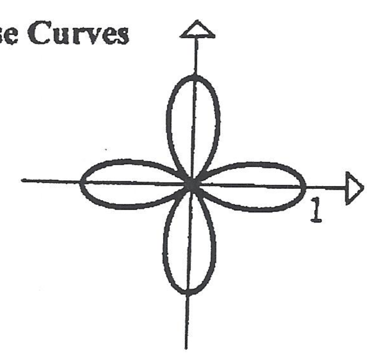


Lemniscate (propeller)

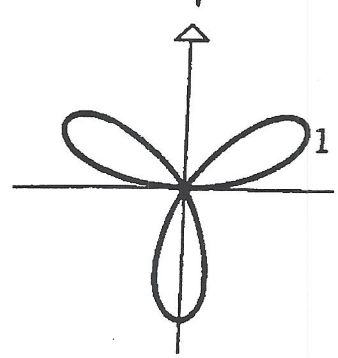
Rose Curves

$r = a \sin (n\theta)$ or $r = a \cos (n\theta)$

If n is odd, then the curve has "n" equally spaced petals.
If n is even, then the curve has "2n" equally spaced petals.



Four-petal rose



Three-petal rose

Polar Curves -- Match the Graph with the Equation

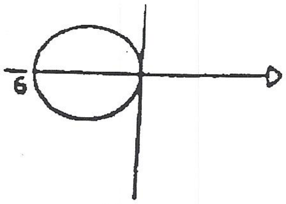
Polar Equations

A) $r = -6\cos\alpha$
B) $r = 3(1 - \sin\alpha)$
C) $r = 1 + 2\sin\alpha$
D) $r = 3 - \cos\alpha$
E) $r = -3 - 4\sin\alpha$
F) $r^2 = \sin 2\alpha$
G) $r = \sin 3\alpha$
H) $2r = \cos\alpha$
I) $r = 4 - 4\cos\alpha$
J) $r = 4 + 3\cos\alpha$
K) $r = 5 + 3\sin\alpha$
L) $r^2 = 9\cos 2\alpha$
M) $r = \cos 2\alpha$
N) $r = 9\sin 4\alpha$
O) $r = -3\sin\alpha$
P) $r = 2 + 2\cos\alpha$
Q) $r = 1 - 2\cos\alpha$
R) $r = 2 + \sin\alpha$
S) $r = 5 - 2\cos\alpha$
T) $r^2 = -16\sin 2\alpha$
U) $r = 2\cos 3\alpha$
V) $r = 1 + \sin\alpha$
W) $r = -5 + 5\sin\alpha$
X) $r = 3 + 2\sin\alpha$
Y) $r = 3 + 4\cos\alpha$
Z) $r^2 = -9\cos 2\alpha$
AA) $r = 3\sin 2\alpha$
BB) $r = \cos 5\alpha$

Answers

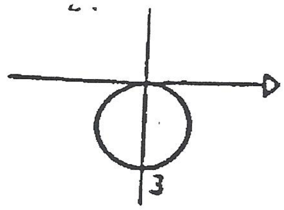
1. A
2. O
3. H
4. V
5. B
6. P
7. I
8. W
9. C
10. Q
11. J
12. X
13. D
14. R
15. K
16 Y
17. E
18. S
19. L
20. Z
21. F
22. T
23. M
24. AA
25. G
26. U
27. N
28. BB

1.



Circle

2.



Circle

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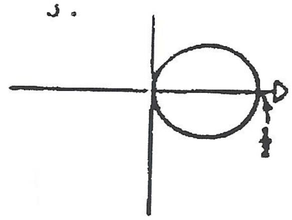

Circle

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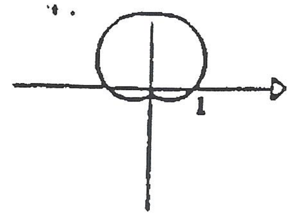

Cardioid

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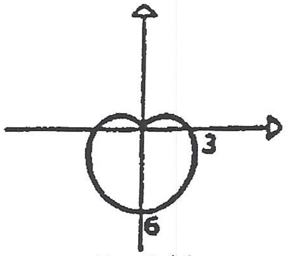

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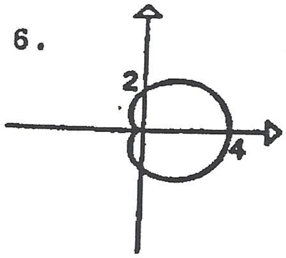

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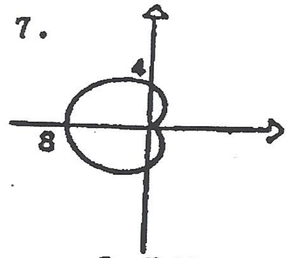

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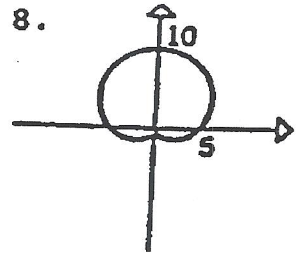

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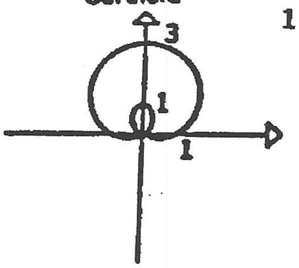

Limaçon

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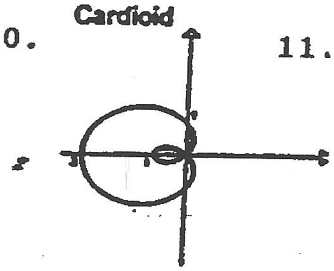

Limaçon

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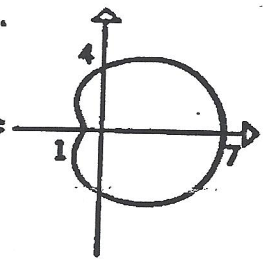

Limaçon

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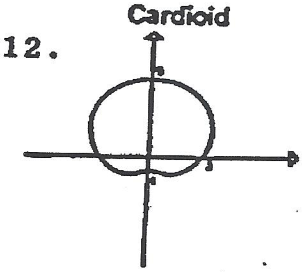

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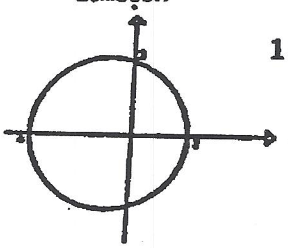

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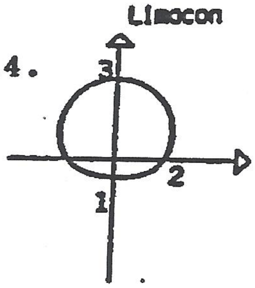

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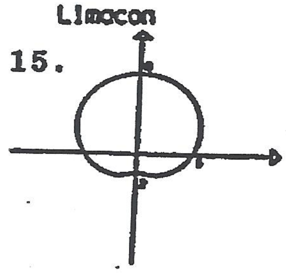

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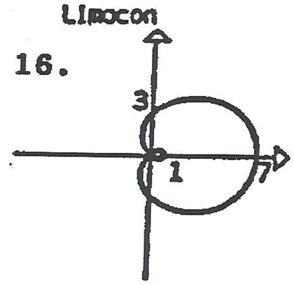

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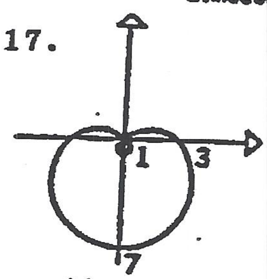

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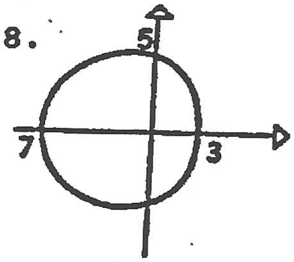

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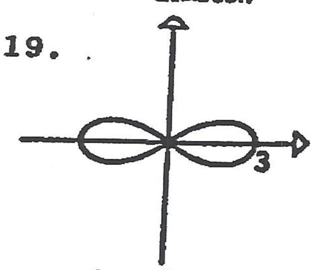

Lemniscate

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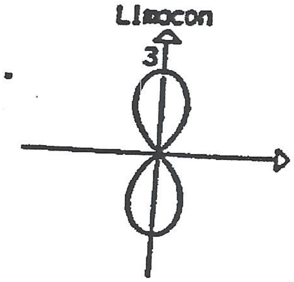

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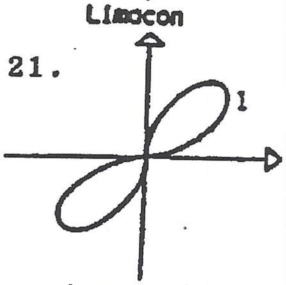

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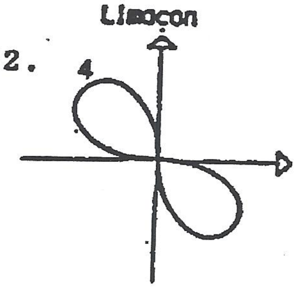

Lemniscate

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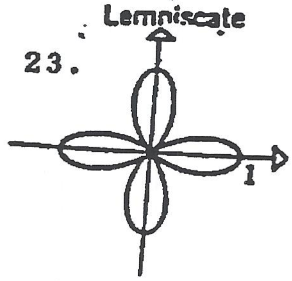
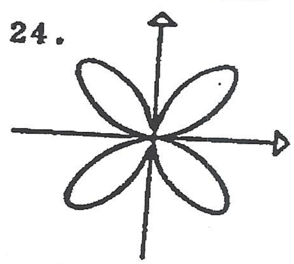


Four-petal rose

24.



Four-petal rose

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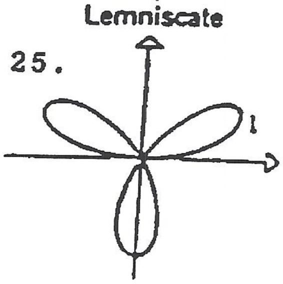
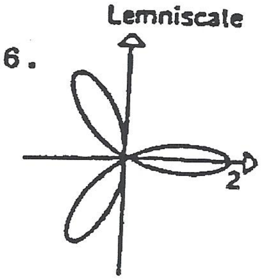


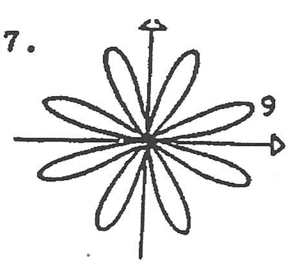
Three-petal rose

26.



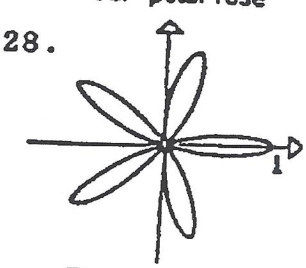
Three-petal rose

27.



Eight-petal rose

28.



Five-petal rose